Fully Ionized Pinch Collapse

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A fully ionized plasma is assumed. To this plasma cylindrically-symmetric magnetic fields are applied, thus causing a pinch collapse. The plasma is treated in hydromagnetic approximation, including electric and thermal conductivity. Separate temperatures are assigned to the electrons and ions.

Two schemes are developed for solving numerically the resulting system of six partial differential equations: the explicit scheme for rather fast pinches, where a numerical stability requirement causes the timestep to be bounded by the characteristics given by the Alfvén speed, and an implicit scheme, which consists essentially in converting the momentum equation into a second order difference equation with coefficients determined by iteration; here there is no such restriction on the timestep. These schemes were made to work on the U.K.A.E.A. IBM 704 and IBM 709.

A run is described in which the initial state was one with uniform density, temperature and B^z field. The boundary temperatures were assumed to remain constant, while the magnetic fields at the boundary were determined by the circuits for the j^z and j^Θ currents. The results of the computations are in good agreement with experimental results obtained at the Technische Hochschule München by one of the authors (Köppendörfer).

The whole program is a joint effort between A.E.R.E. Harwell and the Max-Planck-Institut, intended to discover by comparison with experiments how good the hydromagnetic approximations are. If the agreement is satisfactory (eventually using a generalised program which includes neutral gas) it should be possible to design experiments so that specified field configurations are set up.

This paper treats the simplest possible model for the dynamical problem of the pinch collapse, namely a fully ionized plasma. It is the first of a series of papers dealing with calculations on the pinch. The series will describe a joint attempt by A.E.R.E. Harwell and the Max-Planck-Institut für Physik und Astrophysik, to set up quite general programs for big computers to calculate the pinch effect in hydromagnetic approximation. Within this restriction the equations should be as close as possible to the physics of the actual experiments. The comparison between experiments and these computations should give evidence on how good the hydromagnetic approximation is. It is planned to generalize this program in some respects, e. g. by including neutral gas, anisotropic pressure, toroidal geometry. Eventually one would hope to be able to design circuits to achieve field distributions specified beforehand, e. g. for reasons of stability.

The paper consists mainly of three parts. In the first part the set of equations is derived and the boundary conditions are discussed. In the second part the numerical methods are described. The third part gives the comparison between experiments and computations.

I. The hydromagnetic equations (two fluid model)

A fully ionized plasma is assumed to fill an infinitely long cylinder. Only z and Θ magnetic field components are allowed. All quantities depend only on r, the radius. It is obvious that by this symmetry condition any hydromagnetic instabilities are excluded by definition. A two fluid model with quasineutrality is proposed. Ohmic heating is confined to the electrons, and is determined by the electric resistivity. Shock heating applies only to the ions. Heat is exchanged between these two components at an equipartition rate which depends on electron temperature. The electrical resistivity may be different parallel and perpendicular to the magnetic field. Individual thermal conductivities are included for the ions and electrons, perpendicular to the magnetic field.

The stream velocity of the gas has only one component, in the r direction, which is called shortly v.

The assumption of quasineutrality gives for the particle densities

$$n_e = n_i = n\,. \tag{1.1}$$

Then it follows with the help of the continuity equations for ions and electrons that

$$v_e = v_i = v\,. \tag{1.2}$$

The mass density is

$$\varrho = m_i\, n\,, \tag{1.3}$$

since the elctron mass is neglected.

Then the continuity equation for the plasma is

$$\frac{\partial \varrho}{\partial t} + \frac{1}{r}\,\frac{\partial}{\partial r}\,(r\,\varrho\,v) = 0\,. \tag{1.4}$$

The momentum equation can be written as

$$\varrho\left(\frac{\partial v}{\partial t} + v\,\frac{\partial v}{\partial r}\right) + \frac{\partial}{\partial t}\,(p_e + p_i + q_i) = -B^z\,\frac{\partial B^z}{\partial r} - B^\Theta\,\frac{1}{r}\,\frac{\partial}{\partial r}\,(r\,B^\Theta) \tag{1.5}$$

where p_e, p_i are the electron, ion pressures respectively, and q_i is the artificial shockwave term discussed by RICHTMYER [1] (p. 208)

$$q_i = A\,\varrho\left(\frac{\partial v}{\partial r}\right)^2 \quad \text{if} \quad \frac{\partial v}{\partial r} < 0\,,$$
$$q_i = 0 \quad \text{if} \quad \frac{\partial v}{\partial r} \geq 0\,. \tag{1.6}$$

A is a constant, of the order of magnitude $(\Delta x)^2$, where Δx is the width of the space net used for the numerical integration. This term brings in a kind of artificial viscosity, which makes it possible to compute continuously through the shock, conservation laws through the shock being guaranteed. As implied by the index i it is assumed that the shock heating is confined to the ions, whereas the electrons behave adiabatically. The artificial shock term gives rise to a shock which is some meshpoints wide.

Although this technique works well in fluid dynamics, where the temperature jump across the shock is largely independent of the numerical value of the viscosity, it could lead to trouble in plasma physics because the relative heating of ions and electrons must depend on the ratio of viscosity to resistivity. Therefore the program has since been modified to include a physical estimate of the viscosity.

The usual equation of state is assumed; ε being the internal energy

$$\varepsilon = \frac{p}{\varrho}\,\frac{1}{\gamma - 1}\,. \tag{1.7}$$

Quantities T_e and T_i are defined as

$$T_{e,i} = p_{e,i}/\varrho \tag{1.8}$$

proportional to the usual temperature definition. With these definitions 1.5 becomes

$$\frac{\partial v}{\partial t} + v\,\frac{\partial v}{\partial r} + \frac{1}{\varrho}\,(T_e + T_i)\,\frac{\partial \varrho}{\partial r} + \left(\frac{\partial T_e}{\partial r} + \frac{\partial T_i}{\partial r}\right) + \frac{1}{\varrho}\,\frac{\partial q_i}{\partial r} = -\frac{1}{\varrho}\left(B^z\,\frac{\partial B^z}{\partial r} + B^\Theta\,\frac{1}{r}\,\frac{\partial}{\partial r}\,(r\,B^\Theta)\right). \tag{1.9}$$

The equation for the electron temperature is

$$\frac{\partial T_e}{\partial t} + v\,\frac{\partial T_e}{\partial r} = -(\gamma - 1)\,T_e\,\frac{1}{r}\,\frac{\partial}{\partial r}\,(r\,v) + \frac{1}{\varrho}\,\frac{1}{r}\,\frac{\partial}{\partial r}\left(k_e\,r\,\frac{\partial T_e}{\partial r}\right) + (\gamma - 1)\,\frac{1}{\varrho}\,\varepsilon_J - \frac{T_e - T_i}{2\,t_{eq.}}\,. \tag{1.10}$$

The left hand side and the first term on the right hand side are the usual hydrodynamic terms. The next term is the heat conduction term with a coefficient

$$\varkappa_e = \varkappa_{a1\,e}\,T_e^{5/2}/(1 + \alpha\,\tau_e^2\,\omega_L^2) \tag{1.11}$$

where τ_e is the collision time for electrons, ω_L the LARMOR frequency, and $\alpha \approx 1$. Expressed in terms of ϱ, T and B (1.11) becomes

$$k_e = k_{a1\,e}\,T_e^{5/2}\Big/\left(1 + k_{a2\,e}\,\frac{T_e^3\,B^2}{\varrho^2}\right). \tag{1.12}$$

The third term ε_J is the JOULE heating as given by (1.20) with a factor $(\gamma - 1)/\varrho$ due to the fact that the rate of heating is now per particle rather than per cm³. The fourth term gives the heat loss by the electrons to the ions with equipartition time given by SPITZER [2]

$$t_{eq} = \frac{1}{\varrho}\,T_e^{3/2}\,t_{eqa}\,. \tag{1.13}$$

Finally the equation for the ions is

$$\frac{\partial T_i}{\partial t} + v\,\frac{\partial T_i}{\partial r} = -(\gamma - 1)\left(T_i + \frac{q_i}{\varrho}\right)\frac{1}{r}\,\frac{\partial}{\partial r}\,(r\,v) + \frac{1}{\varrho}\,\frac{\partial}{\partial r}\,k_i\,r\,\frac{\partial T_i}{\partial r} + \frac{T_e - T_i}{2\,t_{eq}} \tag{1.14}$$

with

$$k_i = k_{a1\,i}\,T_i^{5/2}\Big/\left(1 + k_{a2\,i}\,\frac{T_i^3\,B^2}{\varrho^2}\right). \tag{1.15}$$

There is some ambiguity about the magnitudes of the various coefficients, when $T_e \neq T_i$. In the run discussed here resistivity and equipartition rate were taken from SPITZER [2]. The electronic heat con-

[1] R. D. RICHTMYER, Difference Methods for Initial Value Problems, Interscience Publishers, New York 1957.

[2] L. SPITZER, Theory of Physics of Fully Ionized Gases, Interscience Publ., New York 1956.

ductivity for zero magnetic field was taken from SPITZER [2] (p. 87), including his corrections δ_r and ε. MARSHALL [3] (p. 68, eq. (7.17)) gives a heat conductivity which seems to apply only to the electronic component, and his result was used for strong fields, interpolation between the two limits being made according to (1.12). The ionic heat conductivity for zero field was taken from MARSHALL [4] (p. 1), and for strong fields from ROSENBLUTH and KAUFMANN [5], interpolation being made according to (1.13). SPITZER's $\log \Lambda$ was taken as 10 and MARSHALL's ψ as 21.9.

In an improved version of the program Λ and ψ are calculated as functions of T_e and ϱ, and the thermal diffusion effects discussed by MARSHALL [3] are taken into account.

The assumption of different resistivity parallel and perpendicular to the magnetic fields results in a symmetric 2-dimensional resistivity tensor (z- and Θ-directions). OHM's law takes the form

$$E_i = \bar{\mu}_{ik}\, j_k \quad \text{or} \quad \mathfrak{E} = \vec{\bar{\mu}}\, \mathfrak{j} \tag{1.16}$$

where E is the electric field in the coordinate system of the moving plasma. MAXWELL's equations then give

$$\mathfrak{j} = \operatorname{curl} \mathfrak{B}$$

neglecting the induction terms and

$$\partial \mathfrak{B}/\partial t = -\operatorname{curl}[\bar{\mu} \operatorname{curl} \mathfrak{B} + \mathfrak{v} \times \mathfrak{B}]. \tag{1.17}$$

The tensor for the resistivity can easily be obtained as

$$\begin{aligned} \bar{\mu}^{\Theta\Theta} &= \mu^{zz} = \mu(1 + \delta\mu\, B^{z^2}/B^2), \\ \bar{\mu}^{z\Theta} &= \mu^{\Theta z} = -\mu\, \delta\mu\, B^\Theta B^z/B^2, \\ \bar{\mu}^{zz} &= \mu^{\Theta\Theta} = \mu(1 + \delta\mu\, B^{\Theta^2}/B^2) \end{aligned} \tag{1.18}$$

with $B^2 = B^{z^2} + B^{\Theta^2}$, $\mu = \mu_{||}$, $\delta\mu = \dfrac{\mu_\perp - \mu_{||}}{\mu}$. (1.19)

Then the JOULE heating term is given by

$$\varepsilon_J = \mathfrak{E}\,\mathfrak{j} = \mu^{zz}\left(\frac{\partial B^z}{\partial r}\right)^2 + 2\,\mu^{\Theta z}\,\frac{\partial B^z}{\partial r}\,\frac{1}{r}\,\frac{\partial}{\partial r}\,(r\,B^\Theta) + \mu^{\Theta\Theta}\left(\frac{1}{r}\,\frac{\partial}{\partial r}\,(r\,B^\Theta)\right)^2. \tag{1.20}$$

The vector equation (1.17) can be then written in the form

$$\frac{\partial B^z}{\partial t} = \frac{1}{r}\,\frac{\partial}{\partial r}\, r\left(\mu^{zz}\,\frac{\partial B^z}{\partial r} - \mu^{z\Theta}\,\frac{1}{r}\,\frac{\partial}{\partial r}\,(r\,B^\Theta) - v\,B^z\right). \tag{1.21}$$

$$\frac{\partial B^\Theta}{\partial t} = \frac{\partial}{\partial r}\left(-\mu^{z\Theta}\,\frac{\partial B^z}{\partial r} + \mu^{\Theta\Theta}\,\frac{1}{r}\,\frac{\partial}{\partial r}\,(r\,B^\Theta) - v\,B^\Theta\right).$$

Boundary conditions at the wall cause some difficulty. If one simply imposes the condition $v = 0$, as in HAIN [6] eq. (12a), the plasma near the wall sees the full electric field and a current flows

$$\mathfrak{j} = \sigma\,\mathfrak{E}. \tag{1.22}$$

The plasma therefore receives energy at the rate σE^2. The rise in temperature increases σ and also lowers the density by expansion; more heat therefore goes into fewer particles and the calculation breaks down when the temperature and electrical conductivity become extremely large, and all the current is confined to a narrow region near the wall. This effect was studied by HAIN [6], who showed that the numerical difficulties could be avoided by introducing thermal conductivity.

Nevertheless the effect is largely non-physical. Low density plasma near the wall cannot remain at rest, but must move with approximately the "velocity of the lines of force" $(\mathfrak{E} \times \mathfrak{B})/B^2$. Then the current

$$\mathfrak{j} = \sigma(\mathfrak{E} + \mathfrak{v} \times \mathfrak{B}) \tag{1.23}$$

can be very small, since the two terms in the brackets nearly cancel. Two alternative models seem to be possible:

(i) *No plasma emitted from the wall.* According to magnetohydrodynamics the plasma surface will detach itself from the wall, and persist as a sharp boundary, ("cavitation"). Outside this there will be a vacuum in which no currents can flow, and MAXWELL's equations for free space must be used. B^z and B^Θ are independent in this region; B^z = uniform and $B^\Theta \sim 1/r$. The nature of the plasma boundary has been studied by ROSENBLUTH and KAUFMANN [5] and ROSENBLUTH [7].

[3] W. MARSHALL, Kinetic Theory of an Ionised Gas III. AERE T/R 2419.

[4] W. MARSHALL, Kinetic Theory of an Ionised Gas I. AERE T/R 2247.

[5] M. N. ROSENBLUTH and A. N. KAUFMANN, Phys. Rev. **109**, 1 [1958].

[6] K. HAIN, Calculations of the Pinch in the Hydromagnetic Approximation. Proc. of the Fourth Intern. Conference on Ionization Phenomena in Gases, IV A, p. 843 (Uppsala 1959).

[7] M. N. ROSENBLUTH, Proc. of the Second Intern. Conference on Atomic Energy, Geneva 1958, Vol. 31, p. 85.

(ii) *Plasma continuously emitted from the wall.* As the original plasma moves in, fresh plasma is continuously emitted from the wall at some low density, so that the region outside the main pinch remains a good conductor. This model has been studied by COLGATE et al. [8]. B^z and B^Θ are no longer independent.

Model (ii) has been adopted because it is easier to program, (since the same equations can be used throughout), and because it is thought to be more realistic. When the pinch begins some B^z flux is trapped in the insulating wall of the tube. This is sucked into the plasma as the latter is forced inwards by the rising B pressure, so that B^z at the wall falls exponentially to zero. This effect is demonstrated in Fig. 6 e, ($r = 10$ cm), where very good agreement with experiment is shown.

In practice, plasma outside the main pinch can be formed by ionisation of neutral gas which is either emitted by the wall, or left behind when the collapse begins. It is hoped to treat these processes in a quasi-physical way when the partially ionised program is working. At present, we simply postulate that gas emerges from the wall and is ionised immediately.

It might appear that two boundary conditions are imposed in the program, since both ϱ and v are defined at the wall. This would not be valid with equations (1.4) and (1.5). The situation may be understood more easily if (1.4) is rewritten as

$$\frac{\partial \varrho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\,\varrho\,v) = s(t)\,\delta[r-(R-\varepsilon)] \qquad (1.24)$$

where δ is a DIRAC δ-function. Then new plasma is formed at the rate $s(t)$ per unit wall area, at a small distance ε inside the wall. Clearly the function $s(t)$ can be assigned independently of the single boundary condition imposed at $r = R$.

$s(t)$ ought to be determined physically. At present this cannot be done, either experimentally or theoretically, which is unfortunate because it means that the discharge is to some extent unpredictable. This is particularly true of the current sheath in a stabilised pinch, since the sheath develops in the plasma which was originally close to the wall, or has emerged from the wall in the early stages. Furthermore, it appears to be the outside of the current sheath which ultimately determines stability according to the ROSENBLUTH [7] and SUYDAM [9] analyses, once a reversed B^z has been introduced. Hence the stability of the discharge is also to some extent unpredictable. All that we can do at the moment is to choose an $s(t)$ which seems physically reasonable. However it is not believed that this arbitrariness has much effect on the run discussed below.

From the above discussion, it appears that the boundary condition should satisfy the following requirements:

(a) Plasma must emerge from the wall with a velocity $\mathfrak{v} \simeq (\mathfrak{E}\times\mathfrak{B})/B^2$. Then only small currents can flow. Physically, this velocity should be determined by the main body of the plasma, and by the external circuit, since it is in fact the "velocity of the lines of force"; the pressure and density of the plasma outside the main pinch are usually small enough not to exert any appreciable forces on the rest of the system.

(b) The rate of emission must be chosen so that the density of the "pressureless plasma" is physically reasonable. In the explicit program it is important not to have too low a density, otherwise a very small timestep has to be employed (§ 2). A value 5% of the initial density is usually chosen. In the implicit program there is no such restriction.

Early attempts to fulfill (a) and (b) met with a further difficulty. Emission of plasma by the wall gives an extra pressure $\varrho\,v^2$ acting on the pinch, so that the total pressure is

$$\left[\frac{B^{\Theta^2}+B^{z^2}}{2}\right]_{\text{wall}} + [\varrho(T_e+T_i)]_{\text{wall}} + [\varrho\,v^2]_{\text{wall}}\,. \qquad (1.25)$$

The first term is calculated from the behaviour of the external circuit. The second term is of dubious validity, but it is small and bounded. The third term can cause instability, which arises in the following way:

(i) The extra pressure $\varrho\,v^2$ causes the pinch to move in faster.

(ii) Since the pinch moves in faster, v (wall) is increased, and the extra pressure $\varrho\,v^2$ is enhanced.

This instability was in fact observed in various calculations. It arises from a physical error. If in a small region near the wall, almost stationary neutral gas is turned into plasma moving in with velocity v, there must be a discontinuity in the magnetic

[8] S. A. COLGATE, J. P. FERGUSON and H. P. FURTH, W.C.R.L. 5086.

[9] B. R. SUYDAM, Proc. of the Second Intern. Conference on Atomic Energy, Geneva 1958, Vol. 37, p. 157.

field in order to conserve momentum. Thus

$$\frac{\mathfrak{B}_1^2}{2} = \frac{\mathfrak{B}_2^2}{2} + \varrho\, v^2 \tag{1.26}$$

where $\mathfrak{B}_1$ is the field in the insulator, and $\mathfrak{B}_2$ is the field just inside the plasma. I. e. there must be a current sheath at the wall.

Several methods of introducing this discontinuity were tried, but abandoned in favour of the following simple device, which uses the equation of motion. If all the pressure terms are lumped together, (1.5) can be written as

$$\frac{\partial v}{\partial t} = -\frac{1}{\varrho}\frac{\partial P}{\partial r} - v\frac{\partial v}{\partial r}\,. \tag{1.27}$$

In explicit difference form (§ 2), and for the last space point, (1.27) becomes

$$\frac{V_N^{n+1/2} - V_N^{n-1/2}}{\Delta t} = \frac{-\left(P_{N+1/2}^n - P_{N-1/2}^n\right)}{\frac{1}{2}\left(\varrho_{N+1/2}^n + \varrho_{N-1/2}^n\right)\Delta r} - \frac{V_N^{n-1/2}\left(V_{N+1}^{n-1/2} - V_N^{n-1/2}\right)}{\Delta r} \tag{1.28}$$

(inward motion). Here the upper index represents time, and the lower index space. If we choose $V_{N+1}^{n-1/2} = 0$ for all t, then when the system has settled down so that $V_N^{n+1/2} \simeq V_N^{n-1/2}$

$$P_{N+1/2}^n = P_{N-1/2}^n + \tfrac{1}{2}\left(\varrho_{N+1/2}^n + \varrho_{N-1/2}^n\right)\left(V^{n-1/2}\right)^2, \tag{1.29}$$

which is equivalent to (1.26). For outward motion we suppose that any excess pressure $\varrho\, v^2$ can be taken up by the wall, so that there is no need for a discontinuity in magnetic field; correspondingly the second term in (1.28) is replaced by

$$-\frac{V_N^{n-1/2}\left(V_N^{n-1/2} - V_{N-1}^{n-1/2}\right)}{\Delta r}, \tag{1.30}$$

which is small, since $V_N \simeq V_{N-1}$.

The other boundary conditions are now quite simple. Point N is the inner surface of the wall. T_e and T_i are set constant at $N + 1/2$. $\mathfrak{E}$ is calculated at N from equation (1.23), and fed into the circuit equations, which determine $\mathfrak{B}$ at point $N + 1/2$ for the next time step. The wall density drops according to the arbitrary law

$$\varrho_{N+1/2}^{n+1/2} = \left(\varrho_{N-1/2}^{n-1/2} - \varrho_0\right) e^{-\lambda \Delta t} + \varrho_0 \tag{1.31}$$

for inward motion, (λ being a suitable decay rate), and is determined from

$$\partial\varrho/\partial t = -v\, \partial\varrho/\partial r \tag{1.32}$$

for outward motion.

Very similar boundary conditions hold for the implicit program.

II. Numerical methods [10]

In principle there exist three different methods for solving the system of partial differential equations derived in section I.

(1) The characteristic method. This has a certain advantage because for each time and space point it takes the biggest possible time step but it has been excluded since the use of a different time step for every space point leads to a very complicated program.

(2) The LAGRANGian scheme. In the usual hydrodynamics this has been applied with great success. Here it is not so good, because the magnetic fields vary even in regions where the density is low. One also wishes to use boundary conditions in which matter comes off the wall. This would change the number of space points. There seems no doubt however that a modified LAGRANGian scheme could be used.

(3) The scheme finally adopted is an EULERian difference scheme in which the same time step is taken for the whole range of r values, but the space mesh is non-uniform.

In applying this scheme one immediately encounters great difficulties; on the one hand because of the different (and changing) orders of magnitude of the various coefficients and on the other hand because of the thinness of the current sheath, whose position changes with time. To overcome the last difficulty one uses a variable space net whose density depends on the thickness of the current sheath (and maybe on other parameters too) in the following way.

It seems useful to hold the number of space points fixed. Let N be the number of points used, and Δr_j the width of the space mesh at the point r_j, with

$$r_j = \sum_{k=0}^{j} \Delta r_k$$

(lower indices represent the space net, upper ones the time), then

$$\Delta r = A(t)\, F(y, r)\, \Delta x \tag{2.1}$$

(y symbolizes in the following the whole set of independent variables), $A(t)$ is to be normalized, such

[10] Details are given in K. HAIN, A.E.R.E. R/3383.

that

$$R = r_n = A(t) \int_0^1 F(y, r)\, dx .$$

The procedure is the following. One solves the equations for the time t^{n+1} assuming the solution for t^n. Then computes the new space net for the new time. The variables at these new points are then obtained by linear interpolation. The computations (not shown below) for very thin current sheets show very good results if one takes F of the form

$$F = \left(1 + \alpha R \left[\frac{|j_\Theta|}{|B^z_{1/2} + B^z_{N-1/2}| + \beta_1} + \frac{|j_z|}{|B^\Theta_{N-1/2}| + \beta_2} \right]\right)^{-1} \tag{2.2}$$

where α, β_1 and β_2 are constants. Let ΔB be the change in B in one space-step Δr, within a thin current sheath. Then approximately

$$\Delta r = \frac{R/N}{\alpha R |\Delta B| / |B| \Delta R}$$

or

$$|\Delta B/B| \simeq 1/\alpha N .$$

Typically $N = 40$, $\alpha = 0.15$. In a subsequent modification to the program (2.2) has been generalised to take account of rapid variations in other quantities, e. g. ϱ.

In this way extra space-points are automatically crowded into any region where the physical phenomenon exhibits fine detail, (e. g. current sheath, shock front, edge of the main plasma), and withdrawn from regions in which little is happening (e. g. the central region, at the beginning of the discharge), so that accuracy is maintained without excessive use of machine time.

The other difficulty mentioned above consists essentially in the fact that the order of the system is changed if one crosses out the conduction terms. One can express this in terms of characteristics, each conduction term resulting in a double characteristic along the r line. The number of free boundary conditions, which means the number of variables (or their derivatives) which can be given an arbitrary value, is equal to the number of characteristics which go into the boundary. The number of characteristics and the number of boundary conditions are given by the Table 1.

	characteristics	number of free boundary condition
purely hydromagnetic case	$\lambda_{1,2,3} = v$	
	$\lambda_{4,5} = v \pm \sqrt{\gamma(T_e + T_i) + \frac{B^2}{\varrho}}$	if $v_b \geq 0$ 1 if $v_b < 0$ 4 incoming matter
with electrical resistivity	$\lambda_1 = v$	
	$\lambda_{2,3} = v \pm \sqrt{\gamma(T_e + T_i)}$	if $v_b \geq 0$ 3
	$\lambda_{4,5} = \pm\infty$	if $v_b < 0$ 4
	$\lambda_{6,7} = \pm\infty$	incoming matter
with electrical resistivity and heat conduction	$\lambda_{1,2} = v \pm \sqrt{T_e + T_i}$	4
	$\lambda_{3,4} = \pm\infty$	
	$\lambda_{5,6} = \pm\infty$	
	$\lambda_{7,8} = \pm\infty$	

Table 1.

The heat equation and the equation for the magnetic fields are of the following general type

$$\frac{\partial \mathfrak{y}}{\partial t} = \frac{1}{g_1} \frac{\partial}{\partial r} g_2 \frac{\partial}{\partial r} g_3 \mathfrak{y} - v \frac{\partial \mathfrak{y}}{\partial r} - b \mathfrak{y} \frac{\partial v}{\partial r} + \mathfrak{g} \tag{2.3}$$

where g, b are coefficients, which are possibly matrices. To solve this equation numerically, the stability of the numerical calculations has to be ensured. Then the solution of the difference equation tends to the solution of the differential equation in the limit of $\Delta t \to 0$. If one defines the value of the variable y at some point between t^n and t^{n+1}, then by linear approximation

$$y_j^\varepsilon = \varepsilon\, y_j^{n+1} + (1 - \varepsilon)\, y_j^n . \tag{2.4}$$

The differential equation can be written as a difference equation in the form

$$\frac{y_j^{n+1} - y_j^n}{\Delta t} = \mu \frac{y_{j+1}^\varepsilon - 2 y_j^\varepsilon + y_{j+1}^\varepsilon}{(\Delta x)^2} - v_j^n \frac{y_{j+1}^n - y_j^n}{\Delta x} - b\, y_j^n \left(\frac{\partial v}{\partial r}\right)_j^n + g_j^n . \tag{2.5}$$

By applying a Fourier transformation in space, one derives the amplification matrix here as a factor, (assuming g, b being numbers to simplify the argument):

$$G = \frac{1 - 4(1-\varepsilon)\,\mu \frac{\Delta t}{\Delta x^2} \sin^2 k\,\Delta x - v \frac{\Delta t}{\Delta x}\left(2 \sin^2 \frac{k\,\Delta x}{2} + i \sin k\,\Delta x\right) - b \frac{\partial v}{\partial r} \Delta t}{1 + 4\,\varepsilon\,\mu \frac{\Delta t}{\Delta x^2} \sin^2 k\,\Delta x} . \tag{2.6}$$

The stability is guaranteed if G is smaller than one. It seems that $\sin k x = 1$ is the worst value to be satisfied. By taking absolute values one obtains

$$4\,\mu\,\frac{\Delta t}{\Delta x^2}\left[\left(1-(1-\varepsilon)\,v\,\frac{\Delta t}{\Delta x}\right)-(1-2\,\varepsilon)\,\mu\,\frac{\Delta t}{\Delta x^2}\right]+v\,\frac{\Delta t}{\Delta x}\left(1-v\,\frac{\Delta t}{\Delta x}\right)+(b\,\nabla\,v)\,\frac{\Delta t}{\Delta x}\left(1-\tfrac{1}{2}\,b\,\nabla\,v\,\frac{\Delta t}{\Delta x}\right)\geq 0\,. \tag{2.7}$$

If

$$\mu\,\frac{\Delta t}{\Delta x^2} \gg \mathrm{Max}\left(v\,\frac{\Delta t}{\Delta x},\;b\,\nabla\,v\,\frac{\Delta t}{\Delta x}\right) \tag{2.8}$$

the condition is simply

$$\varepsilon \geq 1/2\,. \tag{2.9}$$

The value $\varepsilon = 1/2$ is chosen for the whole calculation because then the difference equations are one order more accurate in time. It follows then that

$$\mu \geq \frac{v\,\Delta x\left(1-v\,\frac{\Delta t}{\Delta x}\right)+b\,\nabla\,v\,\Delta x\left(1-b\,\nabla\,v\,\frac{\Delta t}{\Delta x}\right)}{4\left(1-\frac{1}{2}\,v\,\frac{\Delta t}{\Delta x}\right)}\,. \tag{2.10}$$

This value of μ gives the limit above which it is permissible to apply the stability condition for the diffusion type equation alone, without considering the whole set of equations. If the transportation by convection and by waves is bigger than by conduction, one should chose the time step according to the stability criteria for the purely hydromagnetic case. It is proposed that Δt should always satisfy

$$v\,\frac{\Delta t}{\Delta x} < 1 \tag{2.11}$$

and if ever

$$\mu \leq \frac{b\,\nabla\,v\,\Delta x}{2} \tag{2.12}$$

one should use a method discussed below. The method of solving equation (2.5) which consists essentially of inverting a matrix which has only three diagonal rows, is given by RICHTMYER[1] (p. 101). The approximate method used in the case where (2.12) is fulfilled is the following.

The equations for the temperature and the magnetic fields are for the form

$$\frac{\partial y}{\partial t} = g_1\,\frac{\partial}{\partial r}\left(g_2\,\frac{\partial\,(g_3\,,\,y)}{\partial r}\right)+D\,y \tag{2.13}$$

where g_1, g_2, g_3 are functions of r and of the variable y, such that the derivatives of g_2, g_3 do not destroy the stability conditions. D contains first order differentiation so that the system without the second derivative with respect to r represents hydromagnetic set of equations (i. e. the set of equations without the conduction terms). In each time step it is possible to represent this in the form

$$y^{n+1} = (1+\delta_1+\delta_2)\;y^n \tag{2.14}$$

where the operator δ_1 corresponds to the conduction terms and δ_2 to the hydromagnetic terms. This is now split into two parts; one first solves

$$\bar{y}^{n+1} = (1+\delta_2)\;y^n\,, \tag{2.15}$$

but taking for boundary conditions those given by the conduction term. (The boundary conditions for $r=0$ are the same for both sets of equations whereas at the wall they are different.) If the conduction term is quite small it influences the boundary conditions, but inside the plasma the solution is given by the hydromagnetic equations.

Having solved equation (2.15) one takes

$$y^{n+1} = (1+\delta_1)\;\bar{y}^{n+1}\,. \tag{2.16}$$

The error is of second order in time. This approximation is such, that one neglects the interaction between conduction and other forms of transport within each time step. The effect on stability is to make the hydromagnetic equations more stable, because (2.16) is a smoothing process. The terms

$$(T_e - T_i)/t_{eq} \tag{2.17}$$

are also treated separately, because it often happens that $\Delta t \gtrsim t_{eq}$. This technique of splitting the equations into separate parts representing the various physical phenomena, and solving them in turn at each time step, has been extended in later versions of the program. It simplifies both the physics and the stability analysis and also allows the program to be rapidly changed when new phenomena are to be included. These advantages outweight the slight loss of accuracy.

Having dealt with the conduction terms, one must now consider the methods for solving the remaining set of hydromagnetic equations. There are two schemes now in use

(1) the explicit scheme,
(2) the implicit scheme.

(1) According to RICHTMYER[1] (p. 205) all quantities except v are taken at half integral space points and integral timesteps; v is taken at integral space points and half integral time steps. It was found useful to apply a transformation, here called LA-

GRANGian transformation, to remove the convection term:

$$\begin{aligned}
\bar{\varrho}^n_{j+1/2} &= \varrho^n_{j+1/2} + \frac{1}{2}\frac{\Delta t}{\Delta_{j+1/2}} v^{n+1/2}_j \left(\varrho^n_{j-1/2} - \varrho^n_{j+1/2}\right) && \text{if} \quad v^{n+1/2}_j \geq 0\,,\\
\bar{\varrho}^n_{j+1/2} &= \varrho^n_{j+1/2} - \frac{1}{2}\frac{\Delta t}{\Delta_{j+3/2}} v^{n+1/2}_{j+1} \left(\varrho^n_{j+3/2} - \varrho^n_{j+1/2}\right) && \text{if} \quad v^{n+1/2}_j < 0\,, \text{ and } \quad v^{n+1/2}_{j+1} \leq 0\,,\\
\bar{\varrho}^n_{j+1/2} &= \varrho^n_{j+1/2} && \text{if} \quad v^{n+1/2}_j < 0\,, \text{ and } \quad v^{n+1/2}_{j+1} > 0\,,
\end{aligned} \tag{2.18}$$

and the back transformation

$$\begin{aligned}
\varrho^{n+1}_{j+1/2} &= \bar{\varrho}^{n+1}_{j+1/2} + \frac{1}{2}\frac{\Delta t}{\Delta_{j+1/2}} v^{n+1/2}_j \left(\bar{\varrho}^{n+1}_{j-1/2} - \bar{\varrho}^{n+1}_{j+1/2}\right) && \text{if} \quad v^{n+1/2}_j \geq 0\,,\\
\varrho^{n+1}_{j+1/2} &= \bar{\varrho}^{n+1}_{j+1/2} - \frac{1}{2}\frac{\Delta t}{\Delta_{j+1/2}} v^{n+1/2}_j \left(\bar{\varrho}^{n+1}_{j+3/2} - \bar{\varrho}^{n+1}_{j+1/2}\right) && \text{if} \quad v^{n+1/2}_j < 0\,, \text{ and } \quad v^{n+1/2}_{j+1} \leq 0\,,\\
\varrho^{n+1}_{j+1/2} &= \bar{\varrho}^{n+1}_{j+1/2} && \text{if} \quad v^{n+1/2}_j < 0\,, \text{ and } \quad v^{n+1/2}_{j+1} > 0\,.
\end{aligned} \tag{2.19}$$

Similarly for T_e, T_i, B^z, B^Θ, but not v. – The difference equation for the velocity is written in the form

$$\begin{aligned}
v^{n+1/2}_j = v^{n-1/2}_j - \frac{\Delta t}{\Delta r_{j+1/2}} \Bigg\{ & \left(T^n_{j+1/2} - T^n_{j-1/2}\right) + \frac{1}{\frac{1}{2}\left(\varrho^n_{j+1/2} + \varrho^n_{j-1/2}\right)} \left(q^n_{i,j+1/2} - q^n_{i,j-1/2}\right) - \tfrac{1}{2}\Big[\left(T^n_{j+1/2} + T^n_{j-1/2}\right) \\
& \cdot \left(\varrho^n_{j+1/2} - \varrho^n_{j-1/2}\right) + \left(B^{Zn}_{j+1/2}\right)^2 - \left(B^{Zn}_{j-1/2}\right)^2 + \left(B^{\Theta n}_{j+1/2}\right)^2 - \left(B^{\Theta n}_{j-1/2}\right)^2 \frac{\Delta r_{j+1/2}}{2\, r_j} \left[\left(B^\Theta_{j+1/2} + B^\Theta_{j-1/2}\right]^2\right] \\
& - v^{n-1/2}_j \left(v^{n-1/2}_{j+1} - v^{n-1/2}_j\right) \qquad (\text{if } v^{n-1/2} \leq 0) \\
& - v^{n-1/2}_j \left(v^{n-1/2}_j - v^{n-1/2}_{j-1}\right) \qquad (\text{if } v^{n-1/2} > 0) \Bigg\}
\end{aligned} \tag{2.20}$$

where $T = T_\mathrm{e} + T_\mathrm{i}\,, \qquad D_{j+1/2} = \frac{1}{r_{j+1/2}}\left(r_{j+1}\, v^{n+1/2}_{j+1} - r_{j-1}\, v^{n+1/2}_{j-1}\right) \frac{\Delta t}{\Delta r_{j+1}}\,.$ (2.21)

Furthermore for the mass compression

$$\bar{\varrho}^{n+1}_{j+1/2} = \frac{1 - D_{j+1/2}}{1 + D_{j+1/2}}\, \bar{\varrho}^n_{j+1/2}\,. \tag{2.22}$$

The other compression equations for T_e, T_i, B^z, B^Θ are similar. It can be proved that the stability conditions for this scheme are

$$\Delta t < \mathrm{Min}\left(\Delta r_j / \sqrt{T + B^2/\varrho}\right)_j\,. \tag{2.23}$$

This means that the timestep is bounded by characteristics corresponding to the ALFVÉN speed. In other words, the velocity $\Delta r/\Delta t$ at which influences can propagate across the mesh must be at least equal to the ALFVÉN speed, at which physical influences propagate. The time-step chosen was just 80% of the maximum at each stage. This condition is very restrictive in the outer region of the pinch where the ALFVÉN speed is rather high, because of the low density and the relatively big B field. Furthermore at the edge of this region is the main current sheath, where the step in r must be very small. This scheme seems to be good for fast pinches where big variations occur in times comparable with the time for an ALFVÉN wave to reach the next space step. To get rid of the stability condition the following scheme 2 was developed, (it was not used for the calculation discussed in this paper, but will be used for slower pinches).

(2) Contrary to scheme (1), v was taken at integral time and space steps. The other quantities were taken as before. For simplicity the scheme is explained here for the simple set of hydrodynamic equations in flat space

$$\begin{aligned}
\frac{\partial \varrho}{\partial t} + v \frac{\partial \varrho}{\partial x} + v\, \varrho \frac{\partial v}{\partial x} &= 0\,,\\
\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial x} + (\gamma - 1)\, T \frac{\partial v}{\partial x} &= 0\,,\\
\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + \frac{\partial T}{\partial x} + \frac{T}{\varrho}\frac{\partial \varrho}{\partial x} &= 0\,.
\end{aligned} \tag{2.24}$$

The LAGRANGIAN transformation is applied to all variables and now has the form

$$\overset{L}{\varrho}{}^n_{j+1/2} = \varrho^n_{j+1/2} + \frac{1}{2} v^n_j \frac{\Delta t}{\Delta x}\left(\varrho^n_{j+1/2} - \varrho^n_{j-1/2}\right),$$
$$\overset{L}{T}{}^n_{j+1/2} = T^n_{j+1/2} + \frac{1}{2} v^n_j \frac{\Delta t}{\Delta x}\left(T^n_{j+1/2} - T^n_{j-1/2}\right),$$
$$\overset{L}{v}{}^n_{j+1/2} = v^n_j + \frac{1}{4}\left(v^n_j + v^n_{j-1}\right)\frac{\Delta t}{\Delta x}\left(v^n_j - v^n_{j-1}\right) \tag{2.25}$$

(with suitable changes in the first two equations when $v_j{}^n < 0$, [c. f. (2.18)]). The backward transformation is similar. The differential system can now be written in the following form

$$\overset{L}{\varrho}{}^{n+1}_{j+1/2} - \overset{L}{\varrho}{}^n_{j+1/2} + \frac{\Delta t}{\Delta x}\overset{L}{\varrho}{}^\varepsilon_{j+1/2}\left(\overset{L}{v}{}^\varepsilon_{j+1} - \overset{L}{v}{}^\varepsilon_j\right) = 0,$$
$$\overset{L}{T}{}^{n+1}_{j+1/2} - \overset{L}{T}{}^n_{j+1/2} + (\gamma - 1)\frac{\Delta t}{\Delta x}\overset{L}{T}{}^\varepsilon_{j+1/2}\left(\overset{L}{v}{}^\varepsilon_{j+1/2} - \overset{L}{v}{}^\varepsilon_j\right) = 0,$$
$$\overset{L}{v}{}^{n+1}_j - \overset{L}{v}{}^n_j + \frac{\Delta t}{\Delta x}\Big[\left(\overset{L}{T}{}^\varepsilon_{j-1/2} - \overset{L}{T}{}^\varepsilon_{j-1/2}\right) + \left(\overset{L}{\frac{T}{\varrho}}\right)^\varepsilon_j\left(\overset{L}{\varrho}{}^\varepsilon_{j+1/2} - \overset{L}{\varrho}{}^\varepsilon_{j-1/2}\right)\Big] = 0. \tag{2.26}$$

The upper index ε has the meaning as given in (2.4). The values of ϱ and T at the new timestep are put into the momentum equation. This gives the following second order difference equation for v taking $\varepsilon = 1/2$.

$$\begin{aligned} v^{n+1}_j - v^n_j = &-\frac{\Delta t}{\Delta x}\left[T^n_{j+1/2} - T^n_{j-1/2} + \varrho^n_{j+1/2} - \varrho^n_{j-1/2}\right] \\ &+ \frac{1}{8}\left(\frac{\Delta t}{\Delta x}\right)^2\Big\{\Big[(\gamma-1)\left(T^{n+1}_{j+1/2} + T^n_{j+1/2}\right) + \left(\frac{\tilde{T}}{\varrho}\right)\left(\varrho^{n+1}_{j+1/2} + \varrho^n_{j+1/2}\right)\Big]\Big[\left(v^{n+1}_{j+1/2} + v^n_{j+1/2}\right) - \left(v^{n+1}_j + v^n_j\right)\Big] \\ &- \Big[(\gamma-1)\left(T^{n+1}_{j-1/2} + T^n_{j-1/2}\right) + \left(\frac{\tilde{T}}{\varrho}\right)\left(\varrho^{n+1}_{j-1/2} + \varrho^n_{j-1/2}\right)\Big]\Big[v^{n+1}_j + v^n_j - \left(v^{n+1}_{j-1} + v^n_{j-1}\right)\Big]\Big\} \end{aligned} \tag{2.27}$$

where the mean values are written shortly without indices and the superscripts 'L' dropped. This equation can be solved by the method that has already been used for the conduction equations. It can be easily seen that for $\varepsilon = 1/2$ the amplification matrix is unitary, therefore the difference equation is stable. Taking $\varepsilon = 1/2$ has the further advantage, that the difference scheme is one order higher in time. It is proved in HAIN [10] that the scheme is always unstable for $\varepsilon < 1/2$ and stable for $\varepsilon \gtrsim 1/2$. If one linearised this equation it would become the wave equation for sound waves. In our case the method for solving the nonlinear set of equations is an iterative one. One starts by solving the equations (2.22) for ϱ and T with v^n_j as a first approximation to v^{n+1}_j; then computes with the new values $\varrho^{n+1}_{j+1/2}$ and $T^{n+1}_{j+1/2}$ an approximate value of v^{n+1}_j from (2.23) and returns to (2.22) to compute $\varrho^{n+1}_{j+1/2}$ and $T^{n+1}_{j+1/2}$ once more, etc. The convergence is very good, a factor of about 100 in each iteration step.

In magnetohydrodynamics the situation is more complicated, especially when the conductivity term is large. A detailed description is given in HAIN [10].

III. Comparison with experiments

The purpose of the experiments was to achieve hydromagnetic shockwaves on a linear pinch collapse; therefore it was necessary to make the discharge rather fast. To approach hydromagnetic two-fluid model a strong pre-ionization was used.

If the conductivity of the plasma is high and the gas fully ionized, the mass density must be proportional to the B^z field within the current sheath. By this way the structure of shockwaves could be obtained from magnetic field measurements.

The measurements were made on a linear pinch discharge with an energy of 15 K JOULE. The energy was stored in a capacitor bank of 30 μF and 32 KV. The circuit has a period of 11 μsec producing a maximum curent of 250 to 300 KA. The discharge tube had an inner diameter of 20 cm and a length of 50 cm closed by copper electrodes. The returning conductor had a diameter of 24 cm.

For pre-ionization a smaller discharge with a current up to 20 KA was used. Some care was necessary to get a sufficient homogeneous current distribution all over the tube during pre-ionization. Therefore the pre-discharge could not be made too quickly, on the other hand a long-lasting pre-ionization would produce too many impurities. The main discharge was triggered 12 μs after shooting the pre-heater.

The degree of ionization could only be estimated from the energy going into the plasma and from the shock waves. The existence of shock waves gave some hints of the ALFVÉN velocity and of the degree of ionization. About 40 to 50 percent of the gas was ionized.

The B^z and B^Θ fields were measured by the means of magnetic probes. The probes were usual induction coils with a diameter of 0.8 mm and 40 turns. The wire was 3/100 m thick. The coils were shielded by a slitted brass tube and isolated from the plasma by a quartz tube with a diameter of 2.8 to 3.0 mm. The selfresonant frequency was 15 Mc/sec. To integrate the signal a RC network was used. The probes were gauged in a well known changing field. Although the probe diminishes the conductivity of the plasma in its neighbourhood, the accuracy was not less than 10 percents.

The signals of successive discharges could be reproduced until a short time after the first compression of the plasma. Later on, impurities influenced the discharge without affecting the discharge during first compression within the current sheath.

The series of discharges described here has an initial density of $1 \cdot 10^{15}$ particles per cm^3 and a stabilizing field of 1300 G [11].

A calculation with the parameters used in the experiments was undertaken on the A.W.R.E. (Aldermaston) [12] IBM 709.

The figures give the different quantities as functions of position at different times. In Fig. 1 one sees just the onset of the pinch. The density is already quite low in the outer region, and B^z mainly follows the density. The effect of Ohmic heating brings the electron temperature to a maximum of about 10 eV, whereas the ion temperature lags behind. The current sheath has not moved inwards very far, about 1 cm. In the next Fig. 2 one sees clearly the onset of a shock wave. The matter is driven by the current sheath, whose maximum is indicated by the maximum of the electron temperature. The increase in the temperature is caused to some extent by adiabatic compression but mainly by Ohmic heating by the j^Θ current. The B^Θ field outside falls approximately as $1/r$. The density outside is about 1/20 of the initial density. The mean free path in the 2 cm region ≈ 0.1 cm so that the hydromagnetic approximation

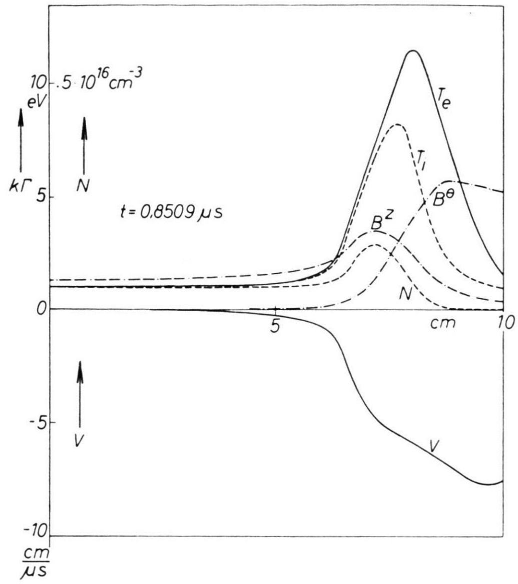

Fig. 1. The calculated quantities versus radius at a time $t = 0.85$ μs after breakdown showing the onset of the pinch.

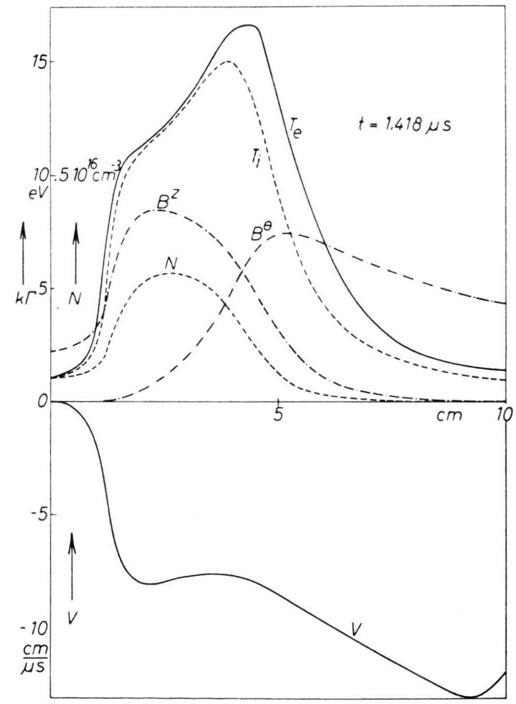

Fig. 2. At this time one sees the onset of a shock wave. The iontemperature T_i in front of the shock reaches the electrontemperature T_e; the ions are heated by collisions, the electrons by the j^Θ current.

[11] Details and results of the measurements with other stabilizing fields and densities will be published by W. Köppendörfer (Z. Naturforschg.).

[12] We should like to thank Dr. J. Corner and Mr. A. H. Armstrong of the Theoretical Physics Division A.E.R.E. for making the computing facilities available to us, and Mr. R. W. Vaughan-Williams for valuable help with the organisation of the program.

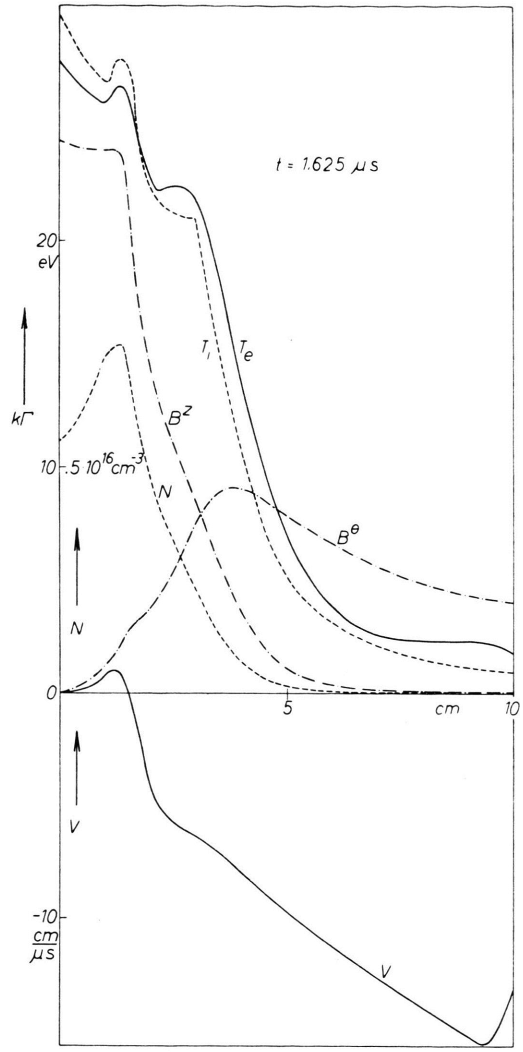

Fig. 3. The pinch at the time of the first maximum compression. The iontemperature is about 30 eV.

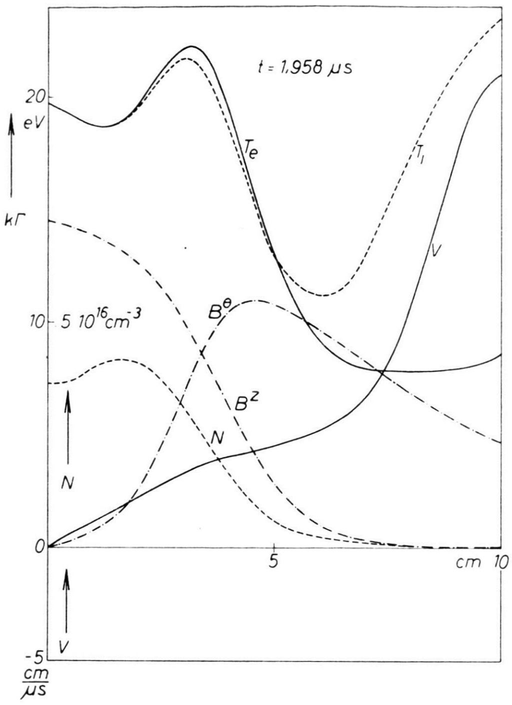

Fig. 4. The reflected shock wave moving outward.

should hold. The next Fig. 3 shows the arrival of the shock wave at the centre. A shock wave is indicated by the fact that the ion temperature is higher than the electron temperature. The compressed B^z field reaches rather high values of about 25 kG in the middle. The peak at 1.9 cm in the temperature is caused by the j^Θ current, and the other one by the j^z current. The last Fig. 4 is some time after the first bounce. One sees the reflected shock wave moving outward, causing a rather high ion temperature, hitting the wall with high speed. This should bring impurities in the next compression. The next printout at about 2.2 μs would show a compression again but this is no longer comparable with experiment because of impurities.

The next series of figures shows the comparison with experiment. The agreement is quite good. The plasma was about 50% ionized in the initial state. Therefore the conductivity was not so good as assumed, hence the magnetic field should be lower. Furthermore, one sees that the experimental data lag somewhat behind in time, which can be explained by the fact, that part of the energy goes into ionization of the neutral gas; this is about one third of the total thermal energy. If one assumes that part of it is also a loss of kinetic energy, then the experiments should be slower on the one hand; on the other hand, if the temperature is not so high the compression should be faster. But since the magnetic fields are quite high this should not be a great effect. Another possible interpretation is that the B^Θ field leaks in, because conductivity is worse than it was theoretically assumed and then it has to compress itself. The effect of slower compression with decreasing pre-ionization could be confirmed by experiments. A comparison of Fig. 5 c and Fig. 5 d is very interesting; the experimental curves show a slope on the inner side which the calculations are

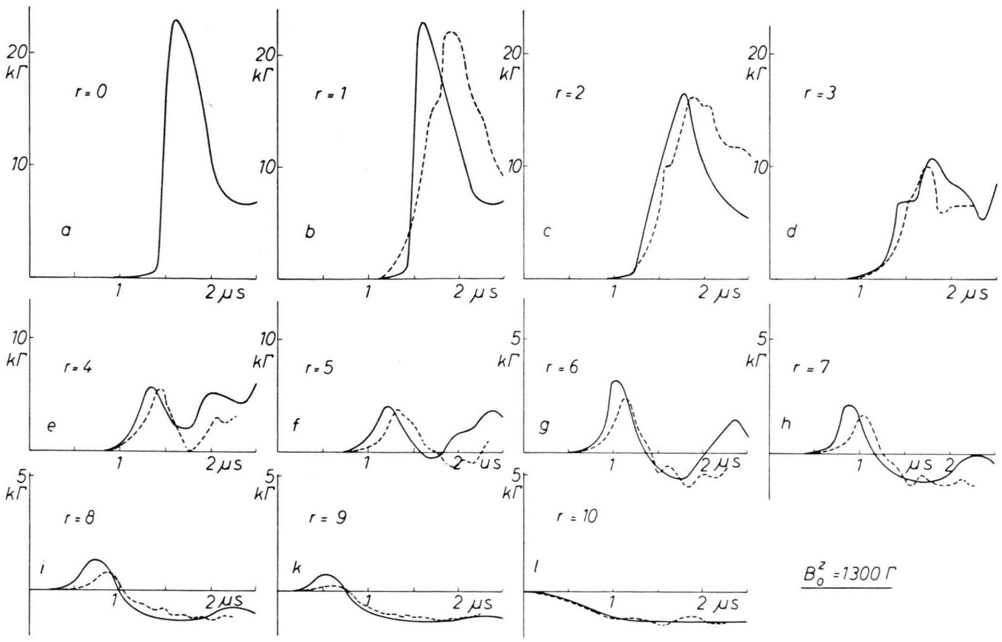

Fig. 5. Comparison between calculated and measured (dashed curves) B_z fields as function of time and different positions. The measured curves were directly copied from oscilloscope pictures.

showing too. A discussion of these slopes is rather difficult, because the B^z field changes in a very short time in magnitude of few 10^{-8} seconds. Surely they are coordinated with shockwaves. Fig. 6 gives the total current.

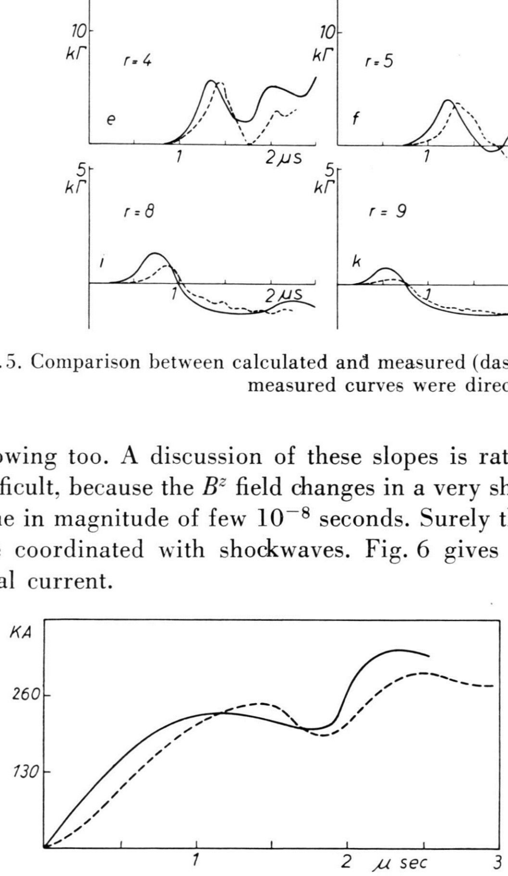

Fig. 6. The total calculated and measured currents. The increase of the measured current is somewhat slower in agreement with the smaller conductivity.

IV. Conclusion

A numerical scheme for computing the dynamical pinch collapse has been set up and made to work on the IBM 704 and IBM 709. The comparison between experiment and computation shows a remarkable agreement, not only in the main profile but also in some details. The width of the computed current sheath compared with the experimental curves indicates a somewhat lower conductivity in agreement with the fact, that the incomplete ionization gives a somewhat higher electrical resistivity. It seems that SPITZER's formula for the conductivity holds. The temperature is in the region of 10 to 15 eV, which seems quite reasonable. The result indicates that the hydromagnetic approximation holds quite well, so that one can hope to gain a better insight into the structure of the pinch by calculating different cases. The main effect of the neutral gas seems to be to slow down the pinch.

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